Some HVI Data Analysis Fundamentals

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Presented at the Fifth Engineered Fiber Selection

Research Forum

November 5, 1992

Raleigh, NC 27612

Abstract

A brief description of analysis, applicable to HVI cotton test data, is presented as an aid or tool that may eventually to the problems of statistical control, deterciting lack of control, and correction in the manufacturing processes. The material covered in this paper has been prepared to improve the general understanding of cotton HVI calibration data, production data, and analytical techniques useful in the assessment and application of that data.

1.Introduction

Scientific methods aid in the solution of problems. The formulation and application of scientific methods are tools in the hands of the skilled scientist. It is useful to understand exactly if, when, and how these "tools" can be applied. What are the assumptions, limitations, and expectations of the methodology and the resulting technology? In the application of analytical techniques to cotton data a wide variety of topics arise:

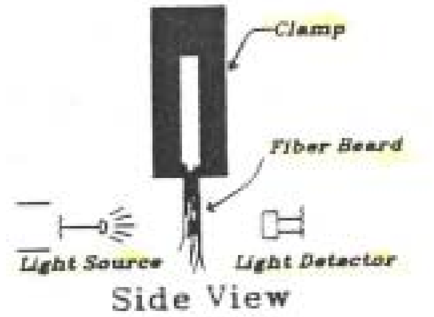
1. data sampling and allocation, experimental design, design philosophy,
2. data analysis, regression analysis, stochastic or statistical results and interpretations,
3. experimental methods, deterministic manufacturing processes, and
4. information utilization, management decisions.

Of course all of these concerns can not be properly addressed in a single paper: instead, this paper focuses on the determination of two fundamental fiber properties, length and strength, which do relate closely to the topics in the above list. Initially, fiber length assessment will be considered through modelling light interaction with cotton fibers to determine the number of fibers in a beard, including the role of randomly sampling fibers of different lengths. Next consideration will be given to a particular aspect of fiber strength assessment, the adjustment of strength values due to the fineness of the fiber as prescribed by Maxwell's Equations applied to the scattering of light by fibers.

The following material considers analyses which may be employed in a decision making mechanism or process applicable to assigning estimates, based on HVI measurement, to the characteristics to be associated with a bale of cotton or group or several bales of cotton.

To begin to understand the content and meaning of HVI data, we must identify or define what is meant by an "HVI value" derived from testing beards extracted form samples consideration, due to differences in measurement techniques for the same or different fiber properties. It is important to examine the underlying physical models which are the origin of the information contained in data and data collection procedures.

2. Modelling Light Interaction With Cotton Fibers To Determine the Number Of Fibers In a Fiber Beard



It is helpful to review certain established relations that help establish the physical content of information contained in a fibrogram generated by the attenuation of light by cotton. The attenuation of photon passage through matter is described by indicating the number of photons removed from the incident beam, using interaction probabilities [1]. The number of photons removed from a bean after passing through matter of thickness dx is :

$$-dN = N_0 \mu dx \quad (2.1)$$

N_0 is the number of photos with no matter present.

Written in terms of, I_0, the initial beam intensity with no matter present, the reduction of photons may be written as:

$$-dI = I_0 \mu dx \quad (2.2)$$

Integration gives

$$I = I_0 e^{-\mu x} \quad (2.3)$$

Then $e^{-\mu x}$ gives the probability that a photon will travel a distance x through the matter without interaction, i.e., the survival probability. μ is the linear attenuation coefficient.

the linear attenuation coefficient equals (atomic cross section) (number atoms/cm^2) with units cm^{-1}.

The mass attenuation coefficient is (atomic cross section) (number atoms/gram), with units $\frac{cm^2}{gm}$, which is the linear attenuation coefficient divided by the density. The number of atoms per gram is Avagadro's number divided by, A, the atomic mass of atoms in the absorber. The atomic cross section equals (Z) (electronic cross section) with Z equal to the atomic number. Replacing the linear attenuation coefficient with the mass attenuation is useful because it depends on the ratio $\frac{Z}{A}$, which is nearly constant from element to element.

μ is a characteristic of the matter, constant for homogeneous matter viewed at constant wavelength. In general the values of these coefficients vary with photon energy. In matter photons lose energy primarily through photoelectric absorption, Compton scattering, and electron-positron pair production. The latter is a higher energy phenomenon which will not be considered further here.

μ may be written as

$$\mu = \sigma_{Compton} + \gamma_{photoelectric} \tag{2.4}$$

Such a description of photon absorption can be extended to formulations, such as, Beer's law [2], a formulation of Equation (2.3) in terms of a concentration of the matter:

$$I = I_0 e^{-\alpha C x} \tag{2.5}$$

Where C is the concentration of the matter and α is a constant, characteristic of the matter. Other expressions may be written for interesting quantities such as photon mean free path. The general description in terms of interaction probabilities lends itself very well to Monte Carlo simulation, making the simulation of the HVI light generated fibrogram test data, not only desirable, but feasible. In particular such an approach may well connect chemical descriptions of cotton with an effective unit structure for cotton, which occurs in the analysis of fibrogram data.

Returning to the interaction description

$$I = I_0 e^{-\mu x} \tag{2.6}$$

we have

$$x_{ij} = \frac{1}{\mu} \ln[\frac{I_{ij}}{I_{i0}}] \tag{2.7}$$

introducing indices for the j^{th} reading in the i^{th} fibrogram, allowing the possibility of evaluating multiple fibrograms.

For homogeneous matter it would be possible to measure μ as the constant slope of the plot of the natural log of the ratio of the measured intensities verses known thicknesses, perhaps determined for cotton by cutting and weighing, assuming constant density of material. In lieu of μ, by observing that, by repetitive sampling procedures, the thickness at a given position is expected to be nearly constant, then μx is nearly constant for each position across samples, with any variation to be accumulated as contributing error in the final estimate. This provides a set of equations in the form

$$\mu x_{ij} = c_j \tag{2.8}$$

The ratio of successive c_j to the maximum c_j provides an evaluation of thickness or amount relative to the largest amount, i.e., the fraction of total fibers at a scan position of length.

The procedure of considering the ratio of each reading to the maximum reading is described as a normalization procedure. This particular normalization is discussed briefly here, but deserves careful consideration. Fixing the normalization condition of fibrogram data is related to an evaluation of the lower limit of fiber length that is assessed in a given sampling method, i.e., short fiber content.

For the arbitrary distribution, consider the ratio of 2 measurements (within the a single fibrogram) of the mass attenuation coefficient.

$$\frac{\mu_{i1}}{\mu_{i2}} = \frac{\frac{1}{x_{i1}} \ln \frac{I_{i0}}{I_{i1}}}{\frac{1}{x_{i2}} \ln \frac{I_{i0}}{I_{i2}}} \tag{2.9}$$

$$\frac{x_{i2}}{x_{i1}} = \frac{\ln \frac{I_{i0}}{I_{i2}}}{\ln \frac{I_{i0}}{I_{i1}}} \tag{2.10}$$

The last expression may be used as the experimental abscissa and scan position as the ordinate in a fibrogram/ for example, the ratio can be applied to provide normalization within a fiber beard scan or collection of beard scans, giving the fraction of total fibers at position relative position 1, the maximum fibrogram reading.

Having reviewed the physical relationships that hold for photon attenuation, length measurements are seen to give the relative thickness at a scan length of cotton (fraction of effectively defined unit fibers, as described in this paper in the discussion of Monte Carlo simulation). The thickness is assumed proportional to the relative number of fibers.

For a data point on the fibrogram, then the ordinate is scan position or length and the experimental abscissa is

$$y_{ij} = \frac{x_{ij}}{x_{i1}} = \frac{\ln \frac{I_{i0}}{I_{ij}}}{\ln \frac{I_{i0}}{I_{i1}}}, \tag{2.11}$$

which reduces to

$$y_{ij} = -C_1 \ln(I_{ij}) + C_2 \tag{2.12}$$

where the constants are

$$C_1 = \frac{1}{\ln \frac{I_{i0}}{I_{i1}}} \tag{2.13}$$

and

$$C_2 = \frac{1}{\ln \frac{I_{i0}}{I_{i1}}} \ln I_{i0} \tag{2.14}$$

One must be careful in applying any normalization relations. Several alternative options are available here. For example, does one extrapolate? Can one extrapolate to some selected value chosen as an intercept corresponding to a choice of origin (minimum length tested) and use that extrapolated intercept value as the maximum? Here again it is seen that the normalization procedure is related to the minim8um length being scanned. When several fibrograms are under consideration, odes one extrapolate using all fibrograms? For several fibrograms does one use the maximum of the group? It is clear that for a group of fibrograms there is the possibility of translating and rotating to best align all fibrograms, then normalizing.

Two sets of two hundred individually normalized fibrograms of U.S.D.A. HVI Calibration Cottons of short and long fiber lengths are shown in sequence in Figures 1 and 2.

3. Randomly Sampling Fibers of Different Lengths

The use of light interaction as a test methodology and the analysis of resulting fibrograms was introduced and extended as a method of fiber length measurement in papers authored or coauthored by Hertel from 1935 to 1960 [3, 4, 5, 6], including discussion of span length as a fiber criteria and factors affecting fiber length scanning measurements. In these papers, Hertel discussed the formulation the fibrogram as a function of a length frequency distribution, to be extracted from the measurement of the length distribution from the measurement of the fibrogram of fiber lengths extending from a clamp remained to be accomplished. Still useful statistics were determined (mean length, span length, upper half mean, etc.).

The fibrogram which results from a clamped random sample of a parent fiber population has been examined by additional current authors, covering alternative test methods [7, 8, 9, 10, 11, 12, 13]. Still the computation of the complete length frequency distribution remained to be accomplished.

Now a direct numerical determination of the fiber length distribution associated with a fibrogram will be presented. Hertel's model for the fibrogram may be expressed as

$$r(l) = \int_{l\,min}^{l\,max} dl \int_{l\,min}^{l\,max} n(l) dl \tag{3.1}$$

where r(l) is understood to be the theoretical abscissa in the fibrogram and n(l) is the number of fibers represented by a column of the length frequency distribution. Hertel's (2.11)) may be numerically inverted for the n(l), giving the length distribution associated with a fibrogram. Three experimentally determined (measured) fibrograms are shown in Figures 3-5. Figures 6-8 show the numerically computed length frequency distributions corresponding to the measured fibrograms. Figures 9 and 10 show two hundred computed fibrograms for the short and long experimental fibrograms shown in Figure 1 and. In addition the cumulative distribution computed from these length distributions and finally the reconstructed fibrograms are given in Figures 11-16.

Investigation of the effects of assuming random fiber sampling may be conducted by examining the resulting length frequency distribution and so may experiment sampling methods be examined in that the right hand side of Equation (3.1) may be modified according to modifications in the sampling method which deviate from random sampling.

A simplified general summary of some current cotton fiber testing concepts and practices follows:

1. assume standardized bale sampling procedure
2. assume random fiber selection for beard preparation
3. assume uniformly defined geometry for an effective unit fiber
4. assume constant density for fiber, say 1.52
5. assume "some certain packing order", area between clamps is constant, number of fibers is constant (a normalization)
6. scan along beard
7. use photon attenuation relation, $I_i = I_0 e^{-\mu x_i}$, to solve for x_i where I_i is measured, I_0 is measured, and μ assumed constant or evaluated
8. number of fibers at scan position x_i or length l is $n(l)$
9. use the fibrogram to generate fiber statistics
10. for strength evaluation, clamp at a predetermined constant x_i, then break measuring the load and elongation along the way.
11. include a strength "adjustment for micronaire"

The methodology outlined in the preceding table demands more elucidation for the resulting data to be useful. It must be made clear that the assumptions are or can be satisfied in practice. Each of the above listed steps deserves further description and examination in the context of the results for computation of the length frequency distribution that has been presented here. Knowledge of this distribution provides the fundamental information required for cotton beard length evaluation.

4. Maxwell's Equations Applied To the Scattering of Light by Fibers

There is an assumption of a continuous homogeneous composition of the matter inherent in the use of the attenuation relation in Equation (2.3). This has been described through the use of an effectively defined unit fiber. If the passage of light through interfiber spaces were negligible, a fiber beard might be interpreted as satisfying these assumption: however, more exact descriptions of the scattering of light from fibers are possible.

An adjustment of optical readings form scattering effects may be employed in many small diameter dependent phenomena, for example the positioning of a clamp to grasp a fiber beard so that it may be broken, may include diameter effect adjustments or corrections.

Micronaire is has been incorporated into this adjustment. A close examination of the origin of this adjustment is desired, to establish the physical origin of the information to be expected from fiber beard strength data. Experimental force displacement curves are shown for both weak and strong cotton corresponding to the two hundred short and long fibrograms shown in Figure 1 and 2.

As a target for light, a single fiber is a partially absorbing material characterized by reflection, refraction, diffraction, and absorption. The reflections, refractions, and absorptions make up the scattering interaction. The diffraction pattern for a fiber is related to a refractive index and diameter of the fiber: minima be related to diameter approximately using the Fraunhofer formula for single slit diffraction [14].

$$\sin\theta = \frac{m\lambda K}{d} \tag{4.1}$$

where m is the m^{th} minima, λ is the wavelength, K is a correction factor for changing application from a single slit to a single fiber (particular to the fiber type, and $d$is the diameter of the fiber. K is also related to d, complication the determination of scattering minima by diameter. Effects of tilted, crimped fibers are possible.)

This description of light scattering by fibers also lends itself to the Monte Carlo type computer simulation as mentioned earlier in the general context of length determination. It is not clear that including a bulk measurement of micronaire provides a suitable determination of fiber diameter to be used as a correction factor for fiber strength [15]. Examination of polarized light scattering may prove to provide this information.

5. Summary

A review of the physical content of HVI length data has been presented in the context of the light attenuation relation, Equation (2.3). Evidence of the direct computation of a single beard length frequency distribution has been introduced. Refinements in beard strength assessment using polarized light have been suggested regarding adjustment for fiber diameter. Alternative investigations (Monte Carlo simulations) have been indicated as feasible for both length and strength.

200 Short Length Fibrograms

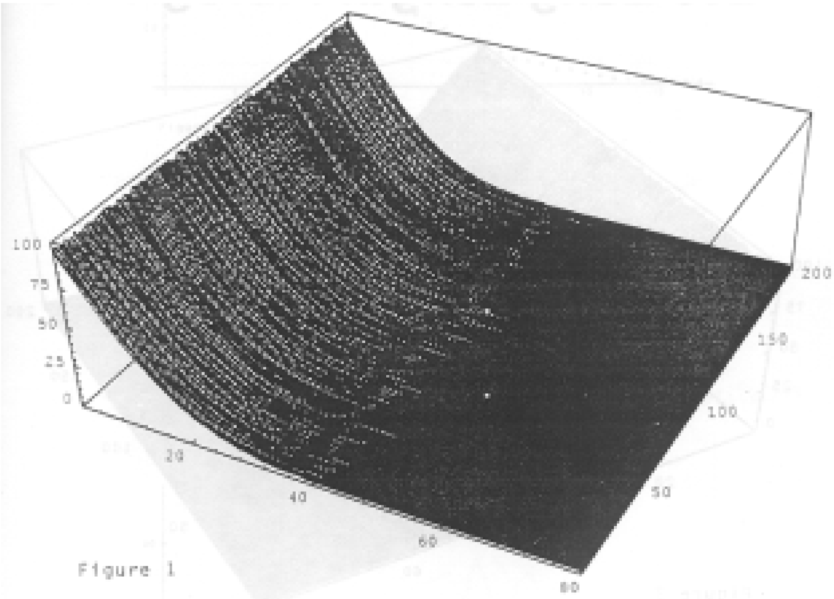


Figure 1.

200 Long Length Fibrograms

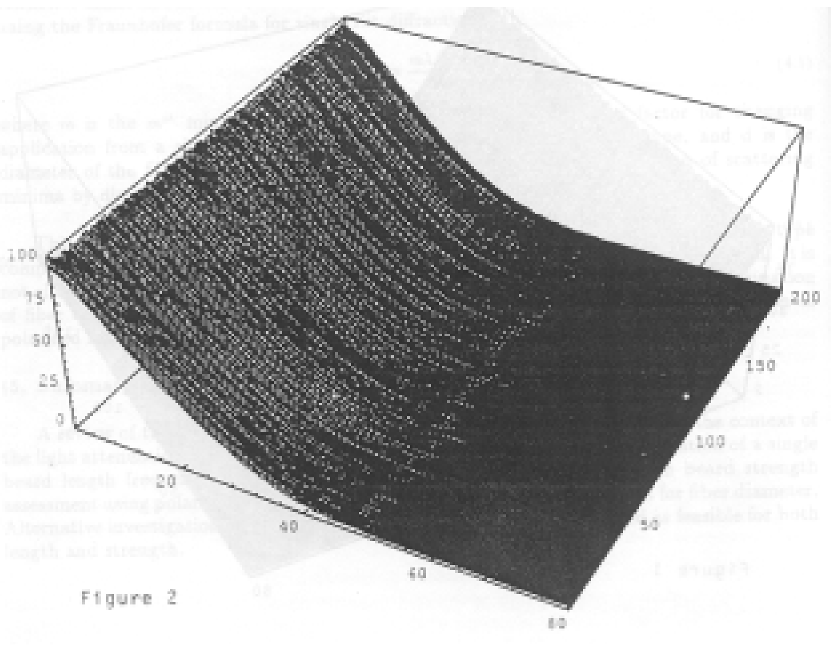


Figure 2. 200 Long Length Fibrograms

Measured Fibrogram 1

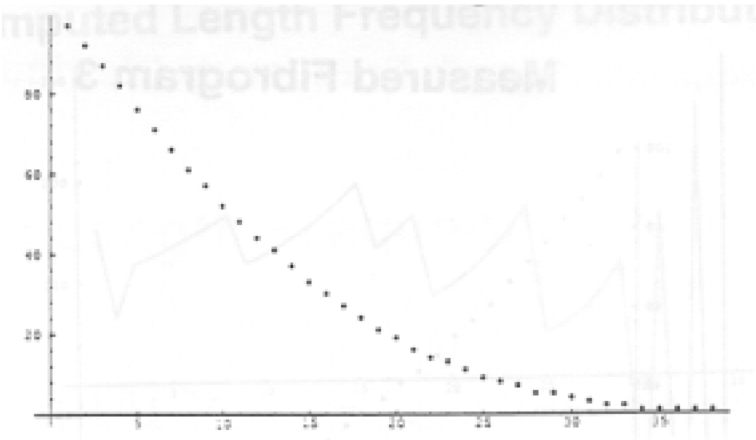


Figure 3. Measured

Measured Fibrogram 2

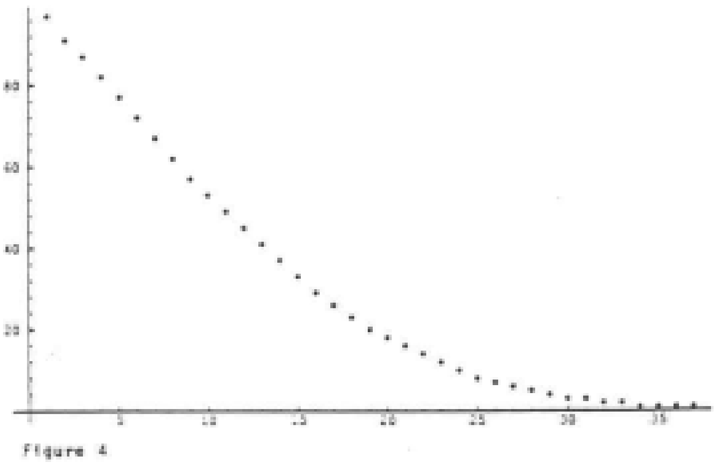


Figure 4.

Measured Fibrogram 3

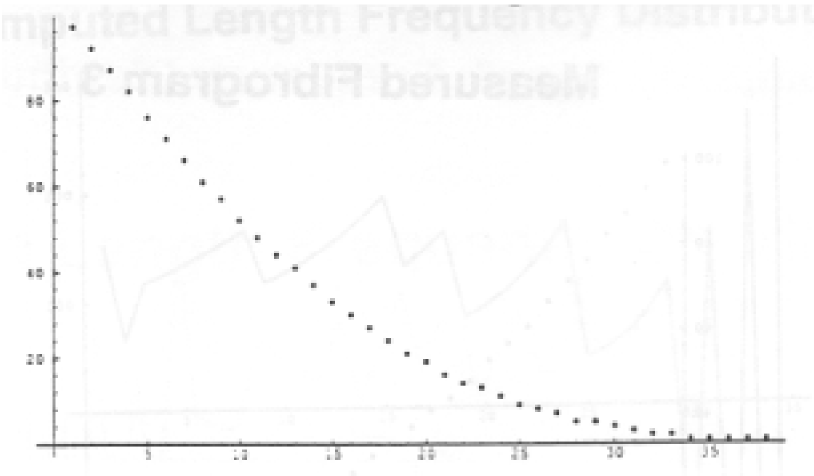


Figure 5.

Computed Length Frequency Distribution

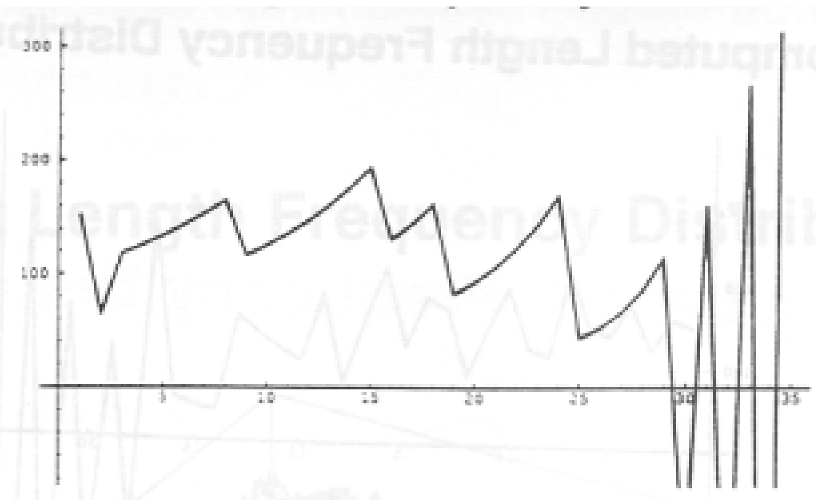


Figure 6.

Computed Length Frequency Distribution

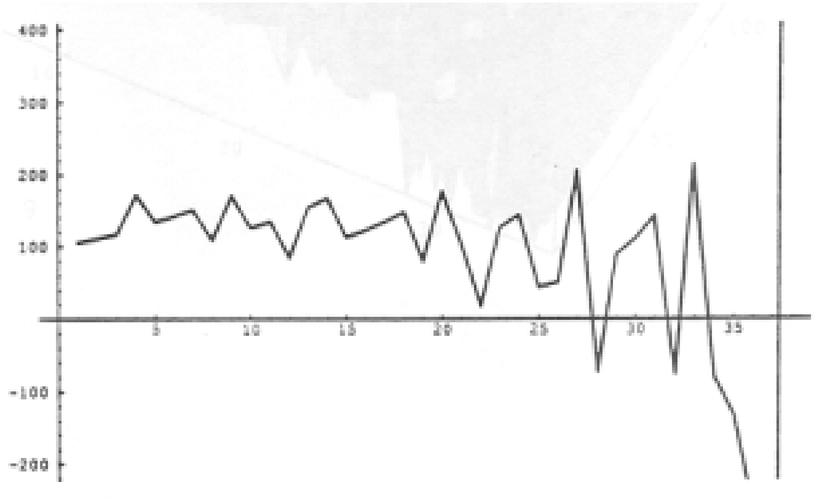


Figure 7.

Computed Length Frequency Distribution

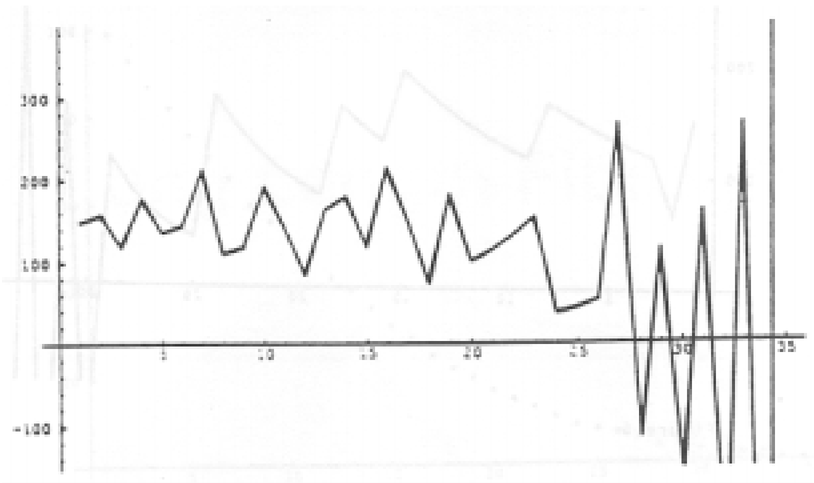


Figure 8.

200 Short Length Frequency Distributions

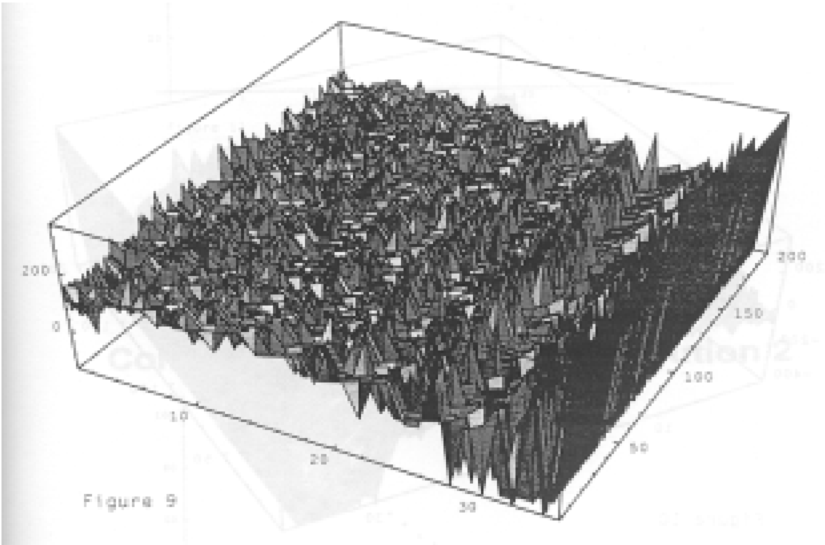


Figure 9.

200 Long Length Frequency Distributions

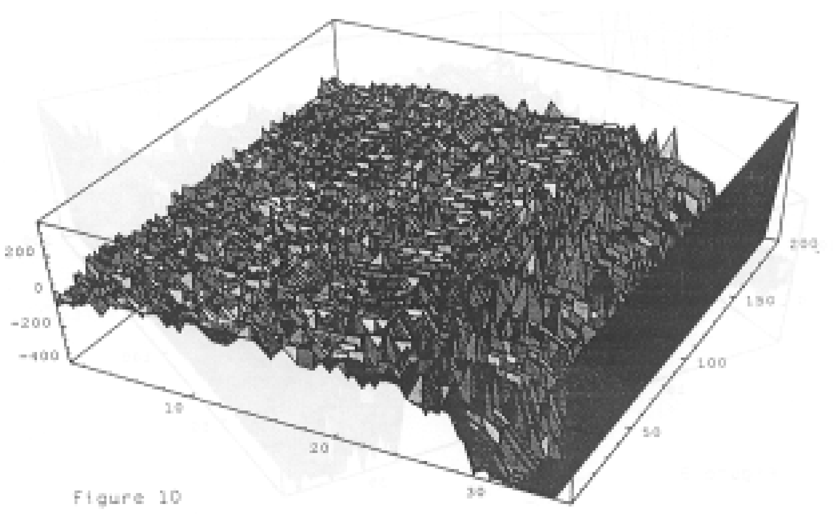


Figure 10.

Computed Cummulative Distribution 1

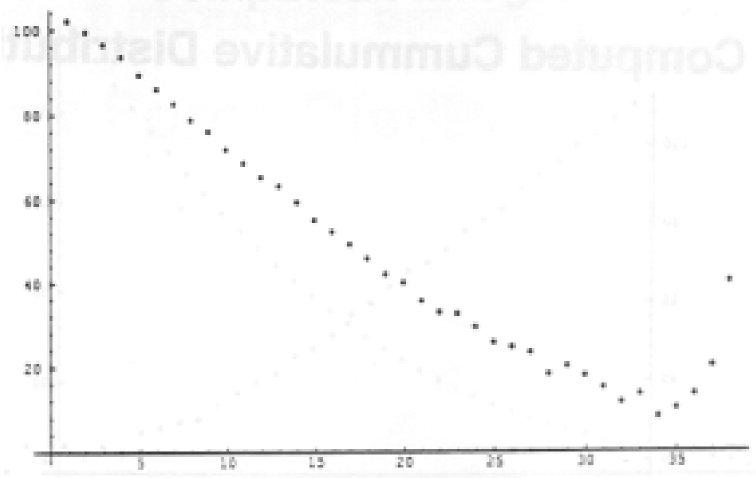


Figure 11.

Computed Cummulative Distribution 2

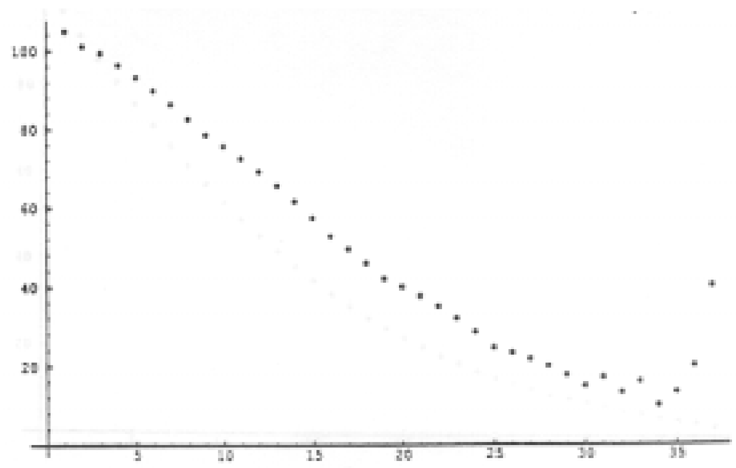


Figure 12.

Computed Cummulative Distribution 3

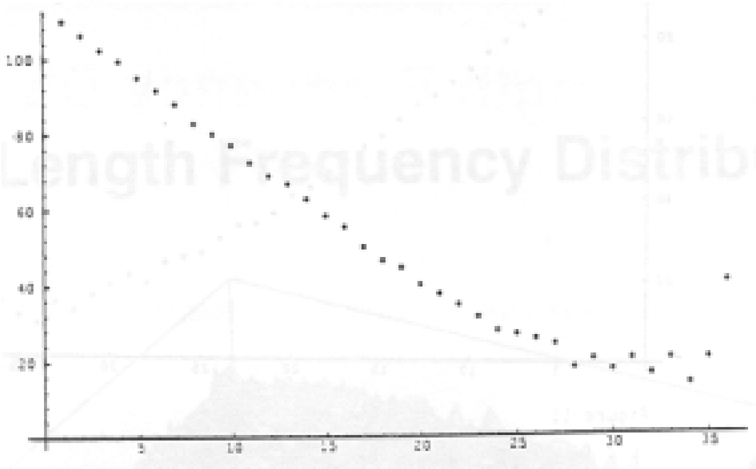


Figure 13.

Computed Fibrogram 1

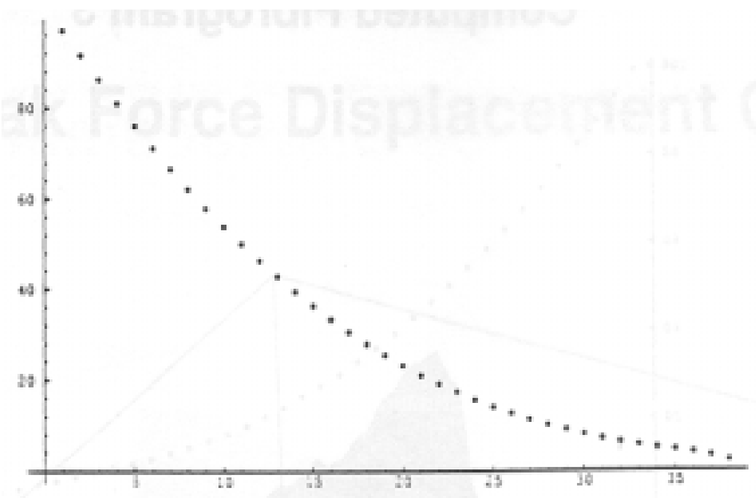


Figure 14.

Computed Fibrogram 2

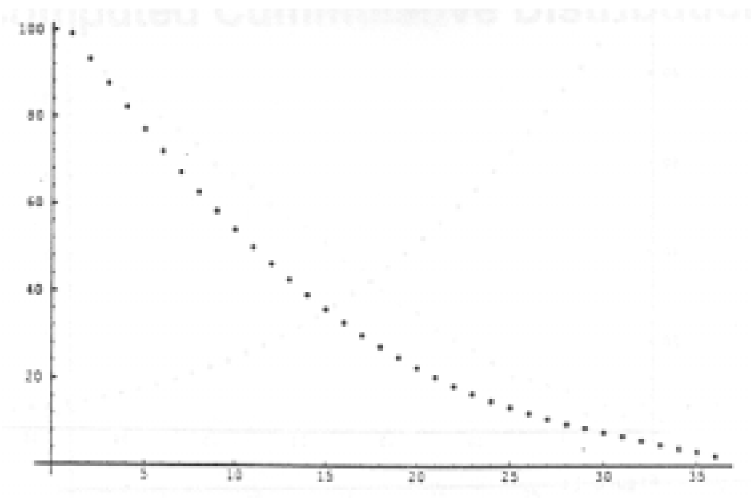


Figure 15.

Computed Fibrogram 3

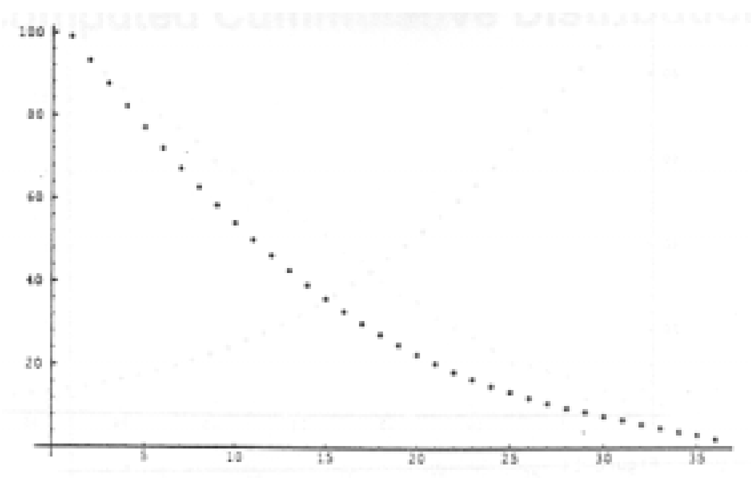


Figure 16.

200 Weak Force Displacement Curves

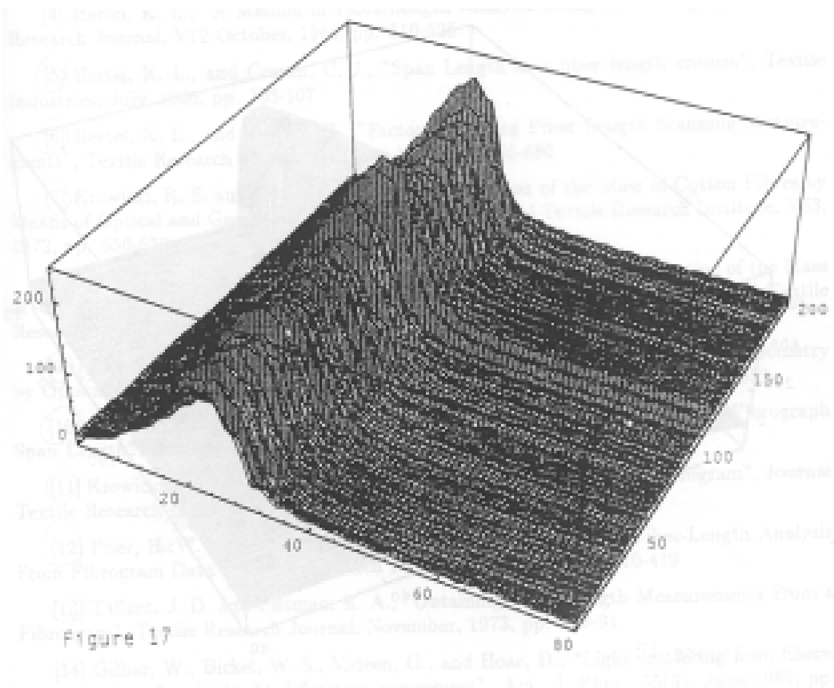


Figure 17.

200 Strong Force Displacement Curves

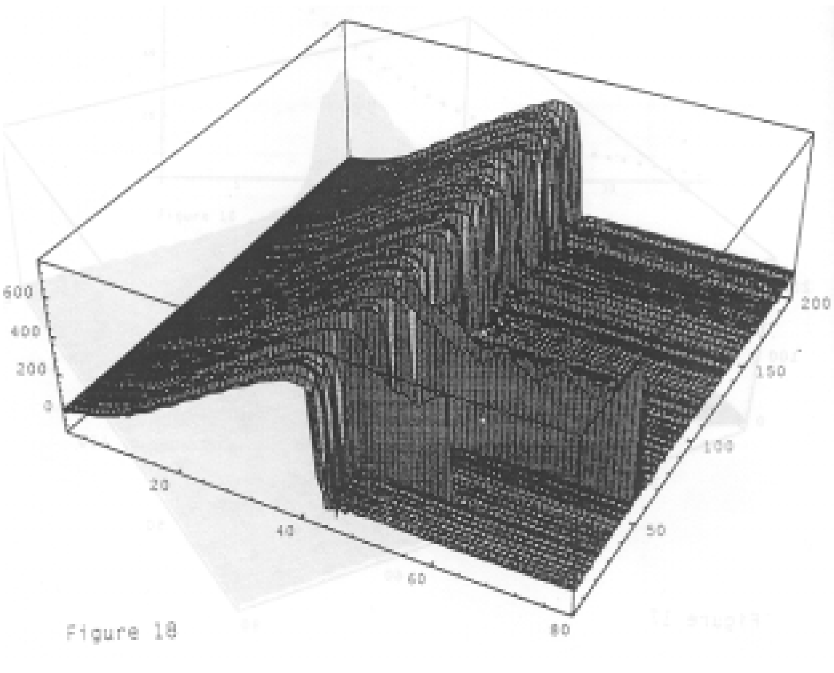


Figure 18.

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